Fabry-Perot-Etalon Plasma Resonator for Faster Operation of the Smart Plasma Antenna

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Abstract— **This paper demonstrates the steering, switching, and reconfiguration speeds in the smart plasma antenna was increased from milliseconds to microseconds. This is done by increasing the plasma density to meet the Fabry-Perot-Etalon conditions. Previously the plasma density was decreased or reduced to zero to allow an open plasma window and a corresponding antenna lobe.**

I. Introduction

The concept of using plasma physics to create antenna array characteristics with or without multiple elements is to surround a plasma antenna or even a metal antenna by a plasma blanket in which the plasma density can be varied [1]-[3]. In regions where the plasma frequency is much less than the antenna frequency, the antenna radiation passes through as if a window exists in the plasma blanket. In regions where the plasma frequency is high, the plasma behaves like a perfect reflector with a reactive skin depth. Hence, by opening and closing a sequence of these plasma windows, we electronically steer or direct the antenna beam into any and all directions. During transmission of the inside antenna of the smart plasma antenna design, the signal can pass through an open plasma window by turning off the plasma in the plasma window or sufficiently decreasing the density of the plasma in the plasma window to allow the signal to pass through. A smart plasma antenna prototype has already been built and demonstrated using these physical principles. A video of this smart plasma antenna can be seen on YouTube [4].

II. Theoretical Model

A faster technique to steer the smart plasma antenna beam, is to increase the plasma density in the plasma tubes so that Fabry-Perot-Etalon effects are met and the signal will pass through. The Fabry-Perot-Etalon effects are well known in optics [5]. The smart plasma antenna can be programmed to meet the Fabry-Perot-Etalon conditions. The characteristic decay time of the plasma after power turn-off is typically many milliseconds, so the opening time of such a barrier generally is predicted also to be many milliseconds. However such a barrier can be opened on a time scale of microseconds by using Fabry-Perot-Etalon effects. This is done by increasing the plasma density rather than waiting for it to decay. The annular cylindrical ring of plasma of the smart plasma antenna creates a Fabry-Perot-Etalon cavity under the proper conditions. When the inside antenna transmits or receives, a standing wave is produced between inside the cylindrical annular ring of plasma with the antenna along the axis. . The Fabry-Perot-Etalon effect for the smart plasma antenna is analogous to the optical Fabry-Perot-Etalon effect. For the smart plasma antenna the effect depends on the boundary layer behavior of the plasma. Once microwave cut-off occurs, one would expect the plasma behavior to be static. What actually occurs is that at microwave cut-off, the reflection is in phase with the incident wave, in analogy to an open coaxial line. (The electron and displacement currents are equal, but out-of-phase) As the plasma density further increases, the reflection smoothly changes from in-phase to 180 degrees out-of-phase, in analogy to a shorted coaxial line in which the reflection current is much greater than the displacement current. The boundary condition at a vacuum-plasma interface of the reflected electric field in terms of the incident electric field is:

$$E_r = \left(\frac{1-i\beta}{1+i\beta}\right)E_0 \tag{1}$$

Where the phase shift is given by:

$$\beta = \sqrt{\frac{\omega_p^2}{\omega^2} - 1} \tag{2}$$

The consequence of this phase shift is that, given any kind of a plasma resonator, if the plasma density is raised high enough, the resonance required for the Fabry-Perot effect to take place must occur. The advantage of this method is that the plasma density can be increased by ionization in microseconds, releasing the microwave radiation. Normally the release of microwave radiation requires the plasma to decay, which occurs on a time of milliseconds. Experiments on Fabry-Perot Effects have been done in a circular plasma resonance cavity. Radiation escaped in experiments even though those plasma tubes were above microwave cut-off. This behavior may be predicted theoretically in that phase of the reflection in the cut-off state varies in phase angle where from 0 degree to 180 degrees causing the resonance in the cavity to form. What this means for the smart plasma antenna application is that microwave emission may be turned on in the ionization time scale of microseconds, rather than in the plasma delay time of milliseconds. When the radius in the annular cylindrical ring of plasma is almost equal by the wave length of the electromagnetic wave, a resonance behavior has been observed. In such case the radiation escapes even those plasma tubes are above microwave cut-off. The resonance frequency changes with the diameter of the circular plasma tubes.

We present a mathematical model for reflection in Fabry-Perot cavity resonator. The general equations may be written as:

$$\varepsilon_0 \nabla . E = \rho \tag{3}$$

$$\nabla \times E = i\omega B \tag{4}$$

$$\nabla \times B = \mu_0 J - \frac{i\omega}{c^2} E \tag{5}$$

$$\nabla . B = 0 \tag{6}$$

By considering $\varepsilon_p = 1 - \left(\dfrac{\omega_p}{\omega}\right)^2$, (7)

where ω_p is plasma frequency, and

$$\beta = \frac{2\pi}{\lambda}, \tag{8}$$

$$k = \beta + j\alpha, \tag{9}$$

$$\frac{d^2 E_z}{dr^2} + \left(\frac{1}{r} - \frac{1}{\varepsilon_p} \frac{\beta^2}{\beta_0{}^2 \varepsilon_p - \beta^2}\right) \frac{dE_z}{dr} + \left(\beta_0{}^2 \varepsilon_p - \beta^2\right) E_z = 0 \tag{10}$$

$$E_r = \frac{\beta}{\beta_0{}^2 \varepsilon_p - \beta^2} \frac{dE_z}{dr} \tag{11}$$

where

$$\beta_0 = \frac{\omega}{c}. \tag{12}$$

Slab Plasma

By considering a slab plasma in the plane xz that plasma exists in $x > 0$, the solution of field equations will be as:

For $x > 0$

$$E(x) = \left(i\frac{k}{x_p}, 0, 1\right) A e^{-x_p x}$$

$$B(x) = \left(0, i\frac{\omega \varepsilon}{c^2 x_p}, 0\right) A e^{-x_p x} \tag{13}$$

For $x < 0$

$$E(x) = \left(-i\frac{k}{x_d}, 0, 1\right) A e^{-x_d x}$$

$$B(x) = \left(0, -i\frac{\omega \varepsilon_d}{c^2 x_d}, 0\right) A e^{-x_d x} \tag{14}$$

where

$$x_p = \sqrt{k^2 - \frac{\omega^2}{c^2}\varepsilon} \tag{15}$$

$$x_d = \sqrt{k^2 - \frac{\omega^2}{c^2}\varepsilon_d}, \tag{16}$$

$$k^2 = \frac{\omega^2}{c^2}\bar{\varepsilon} \tag{17}$$

$$\bar{\varepsilon} = \frac{\varepsilon_d \varepsilon}{\varepsilon_d + \varepsilon}, \tag{18}$$

and

$$\omega_p = \left(\frac{e^2 n}{\varepsilon_0 m}\right)^{1/2}. \tag{19}$$

Then

$$\varepsilon = \varepsilon_r + i\varepsilon_i \tag{20}$$

and

$$\varepsilon(\omega,r)=1-\frac{\omega_p{}^2(r)}{\omega(\omega+i\nu)} \quad (21)$$

we will have:

$$\beta=\frac{1}{\sqrt{2}}\frac{\omega}{c}\left(\sqrt{\bar{\varepsilon}_r{}^2+\tilde{\varepsilon}_i{}^2}+\tilde{\varepsilon}_r{}^2\right)^{1/2}$$

and

$$\beta=\frac{1}{\sqrt{2}}\frac{\omega}{c}\left(\sqrt{\bar{\varepsilon}_r{}^2+\tilde{\varepsilon}_i{}^2}-\tilde{\varepsilon}_r{}^2\right)^{1/2} \quad (22)$$

Where

$$\tilde{\varepsilon}_r=\frac{\left(\varepsilon_r{}^2+\varepsilon_i{}^2+\varepsilon_r\varepsilon_d\right)\varepsilon_d}{\left(\varepsilon_r+\varepsilon_d\right)^2+\varepsilon_i{}^2} \quad (23)$$

$$\tilde{\varepsilon}_i=\frac{\varepsilon_i\varepsilon_d{}^2}{\left(\varepsilon_r+\varepsilon_d\right)^2+\varepsilon_i{}^2} \quad (24)$$

$$x_p=\frac{\omega}{c}\frac{|\varepsilon|}{\sqrt{-\left(\varepsilon+\varepsilon_d\right)}}, \quad (25)$$

$$x_d=\frac{\omega}{c}\frac{\varepsilon_d}{\sqrt{-\left(\varepsilon+\varepsilon_d\right)}}. \quad (26)$$

Then for $x>0$

$$\alpha e^{-x_p x}=$$
$$\exp\left(-\frac{\omega}{\omega_c}\frac{\varepsilon_r}{\sqrt{-\left(\varepsilon_r+\varepsilon_d\right)}}\right) \quad (27)$$
$$\exp\left(i\frac{\omega}{2c}\varepsilon_i\sqrt{\frac{-(\varepsilon_r+\varepsilon_d)}{\varepsilon_r{}^2}}x\right)$$

For x<0

$$\alpha e^{-x_d x}=$$
$$\exp\left(\frac{\omega}{\omega_c}\frac{\varepsilon_d}{\sqrt{-\left(\varepsilon_r+\varepsilon_d\right)}}\right) \quad (28)$$
$$\exp\left(i\frac{\omega}{2c}\frac{\varepsilon_d\varepsilon_i}{\sqrt{-(\varepsilon_r+\varepsilon_d)^3}}x\right)$$

Cylindrical Plasma using modified Bessel Functions.

For $r<R$

$$B_\varphi(r)=B_{r=R}\frac{I_1(x_p r)}{I_1(x_p R)}$$
$$E_r(r)=\frac{kc^2}{\omega\varepsilon}B_{r=R}\frac{I_1(x_p r)}{I_1(x_p R)}$$
$$E_z(r)=\frac{ic^2x_p}{\omega\varepsilon}B_{r=R}\frac{I_0(x_p r)}{I_1(x_p R)} \quad (29)$$

For $r>R$

$$B_\varphi(r)=B_{r=R}\frac{K_1(x_d r)}{K_1(x_d R)}$$
$$E_r(r)=\frac{kc^2}{\omega\varepsilon}B_{r=R}\frac{K_1(x_d r)}{K_1(x_d R)}$$
$$E_z(r)=\frac{ic^2x_p}{\omega\varepsilon}B_{r=R}\frac{K_0(x_d r)}{K_1(x_d R)} \quad (30)$$

$$E_z(r)=E(0)I(0)\left[\left(\beta^2-\beta_0{}^2\varepsilon_p\right)^{1/2}r\right]$$

$$\approx E_z\left(\frac{1-\sqrt{1-\frac{\omega^2}{\omega_p{}^2}}}{1+\sqrt{1-\frac{\omega^2}{\omega_p{}^2}}}\right) \quad (31)$$

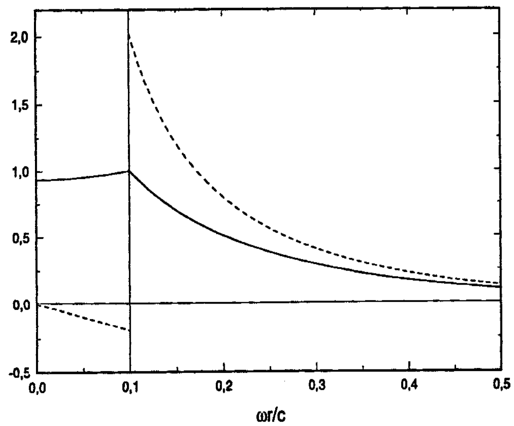

Figure 1. Variation of the components E_z (solid curve) and E_r (dashed curve) as normalized for $\omega / \omega_p = 0.3$

Transverse waves:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{32}$$

$$\nabla \times \vec{B} = \mu_o \vec{j} + \mu_o \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \tag{33}$$

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\nabla \times \frac{\partial \vec{B}}{\partial t} \tag{34}$$

$$\frac{\partial}{\partial t} \nabla \times \vec{B} = \mu_0 \frac{\partial \vec{j}}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{35}$$

$$-\nabla \times \left(\nabla \times \vec{E}\right) = \mu_0 \frac{\partial \vec{j}}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{36}$$

$$-\nabla\left(\nabla \cdot \vec{E}\right) + \nabla^2 \vec{E} = \mu_0 \frac{\partial \vec{j}}{\partial t} + \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{37}$$

$$\nabla^2 \vec{E} = \mu_0 \varepsilon_0 \frac{z e^2 n_e}{\varepsilon_0 m_e} \vec{E} + \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{38}$$

$$\nabla^2 \vec{E} = {\omega_p}^2 \vec{E} + \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \tag{39}$$

$$\vec{E} = \vec{E}_0 e^{i(kx - \omega t)} \tag{40}$$

$$k^2 = \tfrac{1}{c^2}\left({\omega_p}^2 - \omega^2\right) \tag{41}$$

It shows resonance effect when displacement current cancels electronic currnet. $k \to 0$ then $\omega = \omega_p$

When working with a complete ring of plasma tubes with an antenna in the center, we observe a resonance in that microwaves penetrate the plasma ring when the plasma is above cutoff. Such an effect is observed in optics, where it is called a Fabry-Perot Resonator. The question is how can such sharp resonances occur in our resonator? The answer is in the nature of the reflection at the vacuum - plasma interface. If we compute the reflection at the interface, we find the following:

1. At low plasma densities, no reflection occurs.

2. As the plasma density increases so that the plasma frequency approaches the transmitter frequency, reflection gradually increases.

3. When the plasma frequency equals the transmitter frequency, complete reflection occurs.

4. As the plasma frequency increases beyond the transmitter frequency, complete reflection is maintained.

5. As the plasma frequency increases from just equal to the transmitter frequency toward infinity, there is a continuous phase shift from zero to 180 degrees in the reflected signal!

What this means is that during the plasma decay from the initiating pulsed discharge, there will always be a point in which resonance must occur. At this point, the Fabry-Perot effect must occur.

$$E_0 \left(\frac{\left(1 - \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}\right)}{\left(1 + \left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2}\right)} = E_r \tag{42}$$

When the plasma frequency exceeds the transmitter frequency, the above equation becomes complex.

$$E_0 \frac{(1 - i\beta)}{(1 + i\beta)} = E_r \qquad (43)$$

As β increases from 0 to infinity, the magnitude of the reflected signal stays constant. However, the phase of the reflected signal varies smoothly from one to minus one! In electrical engineering terms, at one we have an open line, because the plasma current and the displacement currents cancel. At minus one, we have a shorted line, because the plasma current dominates. This shift of phase in the reflected signal introduces many new possibilities and applications for our plasma antennas! This means that our switching speeds for the storability of the plasma windowing antenna is greatly enhanced because we do not have to reduce the plasma density to get a transmission through it. Our initial tests show that by using the Fabry-Perot Etalon effect we can increase the plasma antenna switching and steering speeds from milliseconds to microseconds.

III. EXPERIMENTAL WORK

A ring of pulsed plasma tubes containing an antenna operating at 3.65 GHz. was tested. The resulting received signal is shown in Figure 2. The plasma when pulsed on, first completely cuts off the microwave signal. As the plasma density increases the signal then rises to complete transmission. However, as the plasma further decays, the signal decreases, and then rises again to complete transmission. We interpret the first transmission peak to the Fabry-Perot-Etalon effect. The effect is completely reproducible, and varies with the transmitter frequency.

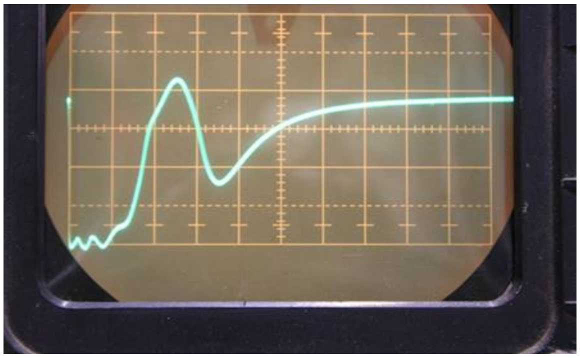

Figure 2.Photo of oscilloscope of Fabry-Perot Etalon experiment.

IV CONCLUSIONS

We have carried out more definitive tests on our Fabry-Perot plasma resonator. The idea is that a ring of plasma tubes containing plasma above microwave cut-off may experience a resonance that allows microwave radiation to escape from the resonator. The advantage of this method is that the plasma density can be increased by ionization in microseconds, releasing the microwave radiation. Normally the release of microwave radiation requires the plasma to decay, which occurs on a time of milliseconds.

We have successfully compared our advanced theory of the microwave plasma Fabry-Perot cavity resonator for faster plasma antenna switching speeds with our previous experimental results and tests.

References

[1] T. Anderson, “Configurable arrays for steerable antennas and wireless network incorporating the steerable antennas.” US patent 7, 342,549, issued March 11, 2008.

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[3] https://www.youtube.com/watch?v=Hu97bwm-OGU

[4] G. Hernandez, (1986). Fabry–Pérot Interferometers. Cambridge: Cambridge University Press. ISBN 0-521-32238-3